Solutions to Problems
FYS3415 – Spring 2026

University of Oslo

Solutions

Problem 1

Let A be Hermitian, so $A = A^\dagger$. Suppose $A|\psi\rangle = \lambda|\psi\rangle$. Then:

$$\langle\psi|A|\psi\rangle = \lambda\langle\psi|\psi\rangle.$$

Taking the conjugate:

$$\langle\psi|A|\psi\rangle^* = \langle\psi|A^\dagger|\psi\rangle = \langle\psi|A|\psi\rangle.$$

Thus $\lambda = \lambda^*$, so λ is real.

Problem 2

A general qubit state is:

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \quad |\alpha|^2 + |\beta|^2 = 1.$$

Multiplying by a global phase $e^{i\theta}$:

$$|\psi'\rangle = e^{i\theta}|\psi\rangle.$$

Measurement probabilities:

$$|\alpha'|^2 = |e^{i\theta}\alpha|^2 = |\alpha|^2,$$

so global phase has no physical effect.

Problem 3

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Then:

$$|0\rangle \otimes |1\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 0 \end{pmatrix}.$$

Problem 4

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle).$$

Probability of measuring $|1\rangle$:

$$P(1) = \left|\frac{1}{\sqrt{2}}\right|^2 = \frac{1}{2}.$$

Problem 5

Assume:

$$|\Phi^+\rangle = (a|0\rangle + b|1\rangle)(c|0\rangle + d|1\rangle).$$

Then:

$$= ac|00\rangle + ad|01\rangle + bc|10\rangle + bd|11\rangle.$$

Matching coefficients with $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$ gives:

$$ad = 0, \quad bc = 0.$$

This forces either $a = 0$ or $d = 0$, etc., which contradicts $ac = bd = \frac{1}{\sqrt{2}}$. Hence no product form exists.

Problem 6

The density matrix:

$$\rho = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|1\rangle\langle 1| = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Problem 7

A quantum channel can be written as:

$$\rho \mapsto \sum_k E_k \rho E_k^\dagger,$$

where $\sum_k E_k^\dagger E_k = I$. Example: bit-flip channel:

$$E_0 = \sqrt{1-p} I, \quad E_1 = \sqrt{p} X.$$

Problem 8

Classical search: $O(N)$.
Grover's algorithm: $O(\sqrt{N})$.

Problem 9

The von Neumann entropy:

$$S(\rho) = -\mathrm{Tr}(\rho \log \rho).$$

It measures quantum uncertainty and generalizes classical entropy.

Problem 10

Violation of Bell's inequality implies that local hidden variable theories cannot explain quantum correlations, demonstrating nonlocality.