FYS3415 / FYS4415

Advanced Quantum Mechanics

Midterm Examination

Date: March 11, 2026
Duration: 2 hours
Allowed aids: Approved formula sheet and calculator
Total points: 100

Instructions:

- Answer all problems.
- Show all steps in your derivations.
- Clearly state assumptions.
- Final answers without reasoning may receive reduced credit.

Name: ____

Problem	Points
1	25
2	25
3	25
4	25
Total	100

Problem 1 (25 points)

Consider a particle of mass m in a one-dimensional infinite square well:

$$V(x) = \begin{cases} 0, & 0 < x < L \\ \infty, & \text{otherwise} \end{cases}$$

(a) Find the normalized eigenfunctions.

(b) Derive the energy eigenvalues.

(c) Compute $\langle x \rangle$ in the state n.

(d) Compute $\langle p \rangle$ in the state n.

Problem 2 (25 points)

Consider the harmonic oscillator Hamiltonian:

$$\hat{H} = \hbar\omega \left(a^\dagger a + \frac{1}{2} \right)$$

(a) Show that $[a, a^\dagger] = 1$ implies equally spaced energy levels.

(b) Compute $\langle n|x^2|n\rangle$.

(c) What is the ground state wavefunction in position space?

Problem 3 (25 points)

Let $\hat{A}$ and $\hat{B}$ be Hermitian operators.

(a) Prove the uncertainty relation:

$$\sigma_A \sigma_B \geq \frac{1}{2} |\langle [\hat{A}, \hat{B}] \rangle|$$

(b) Apply it to x and p.

(c) For which states is the inequality saturated?

Problem 4 (25 points)

Consider a two-level system with Hamiltonian:

$$\hat{H} = \begin{pmatrix} E_0 & \Delta \\ \Delta & E_0 \end{pmatrix}$$

(a) Find eigenvalues and eigenvectors.

(b) If the system starts in state $(1, 0)^T$, compute time evolution.

(c) Find the transition probability as a function of time.